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# Three-Dimensional Finite Element Model for Evaluation the Stress Intensity Factors for Different Fracture Modes of Homogeneous Bimaterial

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*Abstract.* The failure of cracked components is governed by the stresses in the vicinity of the crack tip. The singular stress contribution is characterized by stress intensity factors. The stress intensity factors depend on the geometry of the component and on loading condition. This paper addresses the evaluation of stress intensity factors (SIFs) of opening, shearing and tearing modes for compact tension specimens for bonding homogeneous materials. The aim of this work is to find the optimal pre-cracked length, crack notch radius and crack notch angle in order to eliminate the effect of them on the SIF and then on the Fracture toughness. To achieve this goal, a three-dimension finite element analysis (FEA) model using ANSYS program is constructed for specimens made of homogeneous materials such as stainless steel bonding with epoxy as a filler material. The effects of notch angle, notch tip radius and pre-crack length on the stress intensity factors are studied for different fracture modes. The results for the stress intensity factors $K_I$ , $K_{II}$ and $K_{III}$ are obtained using linear elastic fracture mechanics (LEFM) approach.

*Keywords*: (SIFs) of opening, shearing and tearing modes, FE, Crack notch angle, Precrack length, Crack notch radius.

## 1. Introduction

Bimaterial systems may be found in numerous engineering devices structures and applications. Some of the more common applications include laminated beams coated systems for mechanical devices, and layered composites for main structural applications or micro electronic chips. Most bimaterial systems are constructed by bonding together two similar or dissimilar materials with an adhesive. Other systems may be formed when a material is deposited onto a substrate by means of special deposition techniques. In these bimaterial systems, the fracture behavior at the interface between these dissimilar materials (namely the different layers of the composite) is a critical phenomenon and frequently the weak link in the safe and confident use of these modern materials. Determining the stress intensity factors of interface cracks in bi-materials is the first step in predicting the subsequent crack propagation and damage tolerance. One important point is that the construction with these composite and sandwich systems typically involves the configuration of more or less thin ''strip'' geometry, therefore the formulations and results on infinite plane or half-plane configurations commonly encountered in the literature would not normally be applicable [1].

The fracture behavior of cracked structure is dominated by the near-tip stress field. In fracture mechanics most interest is focused on the stress intensity factors, which describe the singular stress field ahead of a crack tip and govern fracture of a specimen when a critical stress intensity factor is reached. The determination of stress intensity factors for specimens with pre-cracks is important in fracture analysis. The stress intensity factor solutions are reported in handbooks [2-5].

Lee and Keer [6] determined the stress intensity factors for a three-dimensional crack perpendicular to and terminating at a bimaterial interface. It was assumed that the crack is pressurized by a uniform pressure. The method of solution used is based upon the body-force method and requires the analytical results from the elasticity point-force solution for a bimaterial interface. Dubey [7] examined the behavior under tensile load of an elastic plate with a crack. Significance of the general solution of the problem is investigated. It is found that the crack opening parameters can be used as fracture parameters.

Thomas *et al.* [8] considered a center cracked plate subjected to remote tensile and shear loading to determine the stress intensity factors for a circular hole and inclusion using finite element alternating method. Effect of hole and influence of shrunk fit inclusion on the stress intensity factors are studied. William and Robert [9] presented a numerically generated expression to determine crack length in a compact tension specimen from back face strain compliance. Additionally, stress intensity factor and crack mouth opening expressions are determined. The finite element code FRANC2D was used to perform two-dimensional linear-elastic stress analyses on compact tension specimen configurations.

Prime [10] studied the measurement of residual stress and opening stress intensity factors, $K_I$, through the remaining ligament of a compact tension specimen using the crack compliance method with aid of ABAQUS finite element code. The fracture mechanics approach could determine the stress intensity factor caused by the residual stresses with a very simple calculation.

This approach offers the exciting possibility of determining the stress intensity factor prior to a fatigue or fracture test by measuring strains during the specimen preparation.

Hedia and Shabara [11] & Hedia and Fattah [12] studied the effect of notch tip radius and pre-cracking on the stress intensity factors, $K_I$ and $K_{II}$ , for compact tension specimens using two-dimensional finite element model. They built two models one for compact tension specimen without a pre-crack and the other with a pre-crack. For the opening mode, it was concluded that elimination of the pre-crack effect requires that the crack notch radius must be small and the crack notch angle must be less than or equal to $30^o$ [11]. For shear mode, it was concluded that elimination of the pre-crack effect requires that the crack notch radius must be very sharp and the crack notch angle must be less than or equal to $30^o$ [12].

For a crack terminating normally at a bimaterial interface, the stress state in the near-tip region becomes quite complicated. While numerous studies in describing the singular stress behavior for such problems have been performed either analytically or numerically most of the works were considered with the instances subjected to symmetric (mode I) loads. Investigations on study of the asymptotic mixed-mode stress field are therefore in need [13].

The focus of this work is to study the effect of the pre-crack length, a, on the behavior of the opening, shearing and tearing loading modes stress intensity factors in the bonding material (epoxy) which is used as filler material for bimaterial (similar materials such as stainless steel). In addition, the effect of notch angle and notch radius on the opening, shearing and tearing loading modes stress intensity factors are investigated to obtain independent SIF and therefore $K_{IC}$ on the pre-crack length, crack notch radius and crack notch angle; (in other words, to eliminate the effect of pre-crack length, crack notch radius and crack notch angle on the SIF and therefore on the $K_{IC}$ For Bimaterial).

## 2. The Three Dimensional Finite Element Modeling

The finite element method has been used extensively in solving problems involving homogeneous materials. The finite element method has been largely used to analyze various mechanical problems. It has been widely employed for the solution of problems in linear elastic fracture problems.

Figure 1 shows the standard compact tension specimen [9], CT, geometry and defines some of the variables used in the analysis. The specimen size in the finite element analysis was W, with an initial pre-crack length, a. Note that the total crack length, L, is measured from the center of the loading holes rather than from the specimen edge. The modification of compact tension specimen showing the bondline is shown in Fig. 2. The generation of a pre-crack in this specimen is outlined in ASTM 399 [14].

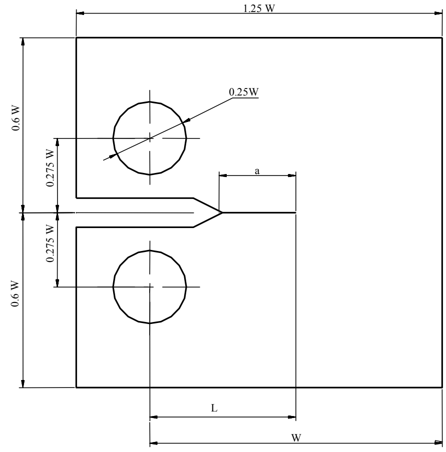



**Fig.1. Configuration of compact tension specimen for fatigue and fracture mechanics-based testing.**

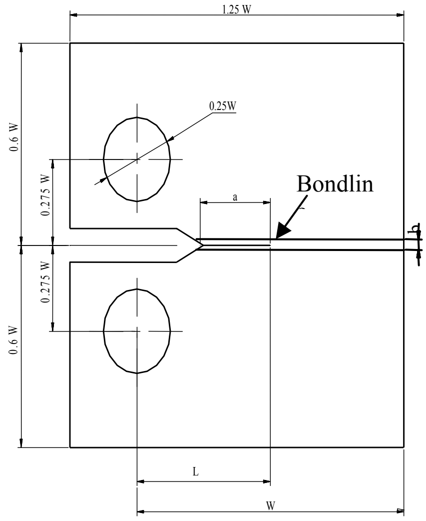



**Fig.2. Configuration of the com- pact tension specimen with centered bonded line and pre- crack.**

The finite element method is used to solve the problem of cracked plates with applied loads and appropriate displacement boundary conditions as in a uniaxial tension test, shear test and tear test device. The finite element program for three-dimensional problems has been used to perform the computations. The recommended element type for the three-dimensional model is SOLID 95. A full specimen is modeled. The first row of elements around the crack front should be singular elements [15].

The specimen has been modeled with two layers through its thickness of twenty node isoparametric hexahedron (brick) elements as shown in Fig.3. In the finite element formulation, the pre-crack of the specimen has been modeled by placing identical independent nodes. The x- and y- axes are in the plane of the plate and the z-axis is perpendicular to the plane of the specimen. The ANSYS code is developed to predict the propagation of fracture.

Full geometry models with a pre-crack are used to measure the stress intensity factors for opening, shear and tear modes. The difference between the models are in the applied loading direction and the boundary conditions. For the opening mode, the applied loads are made to act on the upper and lower surfaces of the specimen as shown in Fig.4. In the shear mode, the applied loads are made to act oppositely at the side of the notch as shown in Fig.5. For the tear mode, the applied loads are made to act oppositely at the surface of the notched specimen as shown in Fig.6.

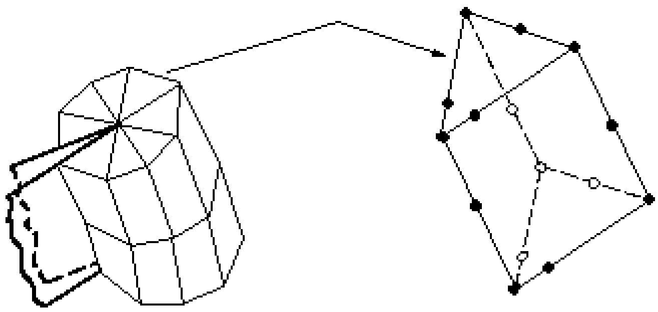
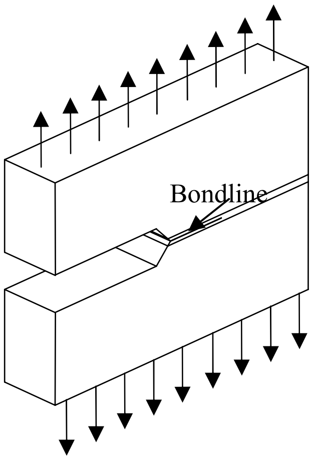


**Fig. 3 . Twenty node isoparametric hexahedron (brick) elements and the first raw elements around the crack tip.**



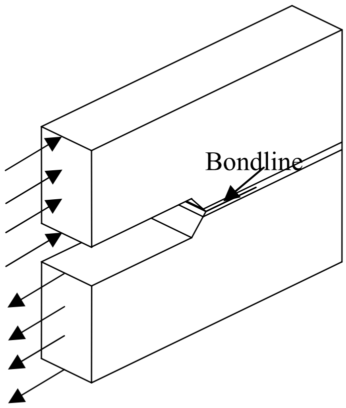




**Fig. 4. The direction of axial pressure applied to simulate the opening mode.**

**Fig. 5. A lateral pressure loading condition on the specimen to simulate the shear loading mode**

For all models, the notch angle varied from $8^o$ to $120^o$. The notch radius, r, varied from 0 (sharp edge) to 0.03333W, where W is the width of the specimen. Also the pre-crack length, a, varied from 0.00667 L to 3.33 L, where L is the total crack length.

The ANSYS input code is written in terms of notch angle, notch radius and pre-crack length in order to change the variables easily. All materials considered were homogenous and linearly elastic. Mesh refinement issues become complicated for three-dimensional models. Fine mesh of size less than 0.0025W are used in modeling the crack tip region. The mesh size is then gradually changed to coarser mesh as the edge is approached.

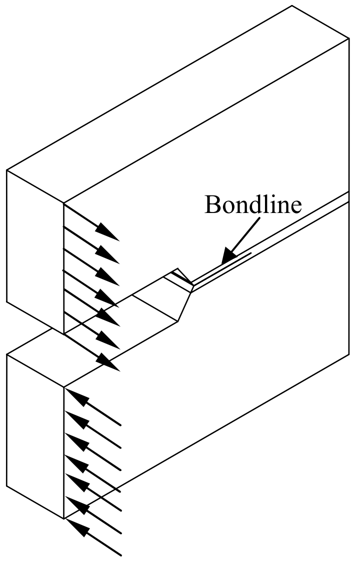



**Fig. 6. The out of plane pressure applied to simulate the tear mode.**

A total of 11892 elements with a total of 27490 nodes were generated in the discretization procedure that for two layer elements through the specimen thickness as shown in Fig.7. The mesh of the model used to measure the stress intensity factor for the opening, shear and tear modes is shown in Fig.7. This mesh includes two layer- elements through the crack tip. The mechanical properties of Epoxy are shown in Table 1.

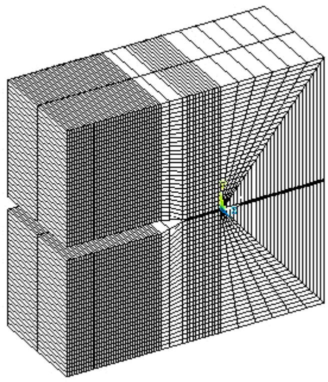


**Fig. 7. Three dimension mesh of the first model includes two layer elements through the crack tip.**

**Table 1. The mechanical properties of Epoxy [16].**

| Materials | Young's Modulus GPa | Tensile Strength MPa | Fracture toughness $MPm^{1/2}$ | Poisson's ratio |
|---|---|---|---|---|
| Epoxy | 2.5 | 35 | 1.2 | 0.3 |

## 3. Results and Discussion

Finite element analysis is briefly explained for stress intensity factors. Mode I, mode II and mode III stress intensity factors ($K_I$, $K_{II}$ and $K_{III}$) are obtained using finite element analysis with the aid of Ansys Parametric Design Language (APDL). The finite element code ANSYS was used to perform three-dimensional linear-elastic stress analyses on compact tension specimen configurations.

The results from the running ANSYS codes, a full three-dimensional model, for bonded homogeneous stainless steel by epoxy as a filler material are shown in Fig. 8 - 16. The results consist of three groups; the first group is concerned with stress intensity factor for a specimen under opening mode loading conditions. The second group is concerned with stress intensity factor for a specimen under shear mode loading conditions. The third group is concerned with stress intensity factor for a specimen under tear mode loading conditions.

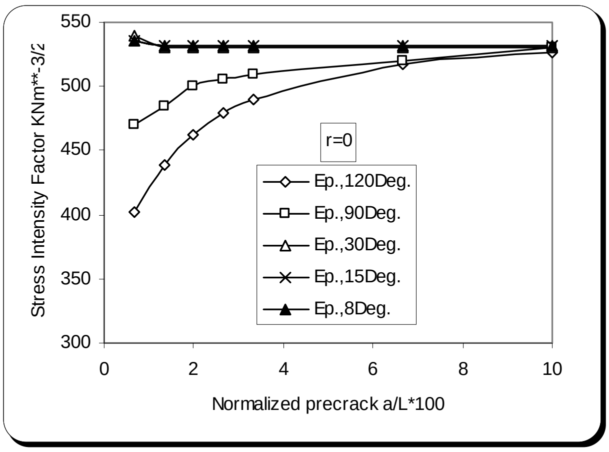



**Fig. 8. Effect of the pre-crack length on the stress intensity factor, KI, for a filler material (Epoxy) for different crack notch angles and r/w ≈ 0.**

### 3.1. Opening Mode Stress Intensity Factor, $K_I$

Figure 8 shows the effect of the pre-crack length on the stress intensity factor, $K_I$, for a filler material (Epoxy) for different notch angles and sharp notch, *i.e.*, notch radius, r, approximately equal to zero. It is observed that the stress intensity factor, $K_I$, increases as the crack notch angle decreases until it reaches a maximum value at an angle equal to 30°. Also, the stress intensity factors remain constant regardless of the value of the pre-crack length, a, greater than or equal to 10% of the total crack length, L. Therefore, the stress intensity factor becomes independent of the pre-crack length beyond a value of a pre-crack of 10% of total crack length.

The stress intensity factor, $K_I$, increases as the pre-crack length increases for all crack notch angles greater than 30° until reaching a maximum value at a pre-crack length, a, greater than or equal to 10% of the total crack length, L.

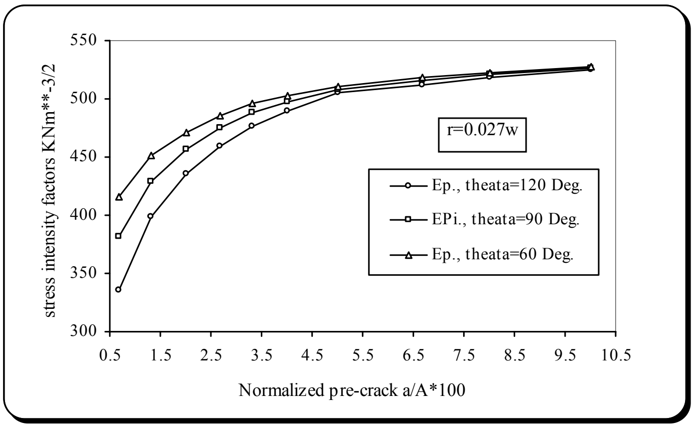



**Fig. 9. Effect of the pre-crack length on the stress intensity factor, KI, for a filler material (Epoxy)for different crack notch angles and r/w = 0.027.**

Figure 9 shows the effect of the pre-crack length on the stress intensity factor, $K_I$, for a filler material ( Epoxy)for different crack notch angles and for a given value of the notch radius, r = 0.027W. The stress intensity factor increases gradually reaching a maximum value at a pre-crack length, a= 0.1L. Also, it is observed that the stress intensity factor becomes not affected by the crack notch radius and is little affected by the crack notch angle for this case.

Figure 10 shows the effect of the crack notch radius on the stress intensity factor, $K_I$, for a filler material (Epoxy) for different pre-crack lengths and crack notch angle equal to 30°. The stress intensity factor, $K_I$, decreases as the crack notch radius increases for a pre-crack shorter than 0.1L. Also, It is observed that the stress intensity factor remains constant for a pre-crack length greater than or equal to 10% L, regardless of the notch radius and notch angle.

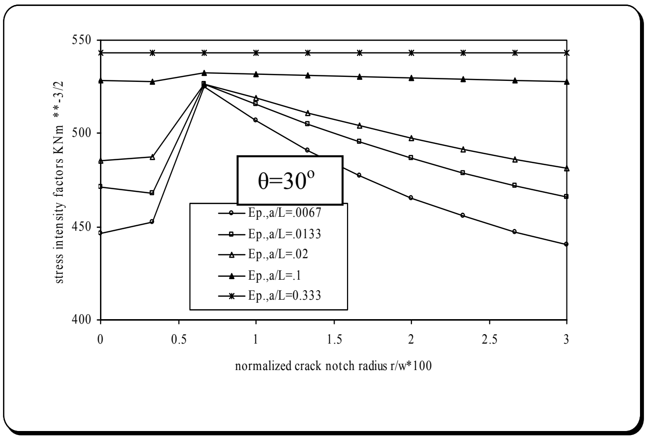



**Fig. 10. Effect of the crack notch radius on the stress intensity factor, KI, for a filler material (Epoxy) for different pre-crack lengths and crack notch angle equal to 30°.**

### *3.2. Shear Mode Stress Intensity Factor, $K_{II}$*

Figure 11 shows the effect of pre-crack length on the stress intensity factor, $K_{II}$, for a filler material (Epoxy) for different crack notch angles and sharp notch, *i.e*., notch radius, r, approximately equal to zero. It is observed that the stress intensity factor increases as the crack notch angle deceases. Also, the shear mode stress intensity factor increases as the pre-crack length increases reaching a maximum value at a pre-crack length greater than or equal to 33% of the total crack length, L, then the stress intensity factor remains constant, *i.e.*, independent on the notch angle.

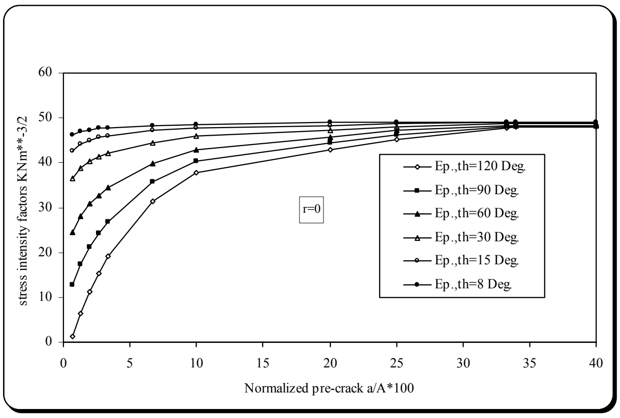



**Fig. 11. The effect of pre-crack length on the stress intensity factor, KII, for a filler material (Epoxy) for different crack notch angles and r/w ≈0.**

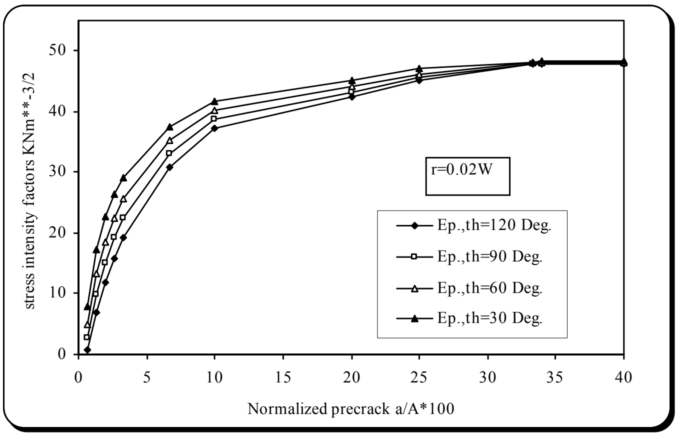



**Fig. 12. Effect of pre-crack length on the stress intensity factor, KII, for a filler material (Epoxy) for different crack notch angles with a blunt notch r = 0.02w of specimen width w.**

Figure 12 shows the effect of pre-crack length on the shear mode stress intensity factor for a filler material (Epoxy) for different crack notch angles with a blunt notch, r, equal to 0.02 of specimen width, W. The shear loading mode stress intensity factor increases as the pre-crack length increases reaching a maximum value at a pre-crack length, a, greater than or equal to 33% of the total crack length, L. However, for the blunt notch, the shear loading mode stress intensity factor is not affected by the crack notch angle.

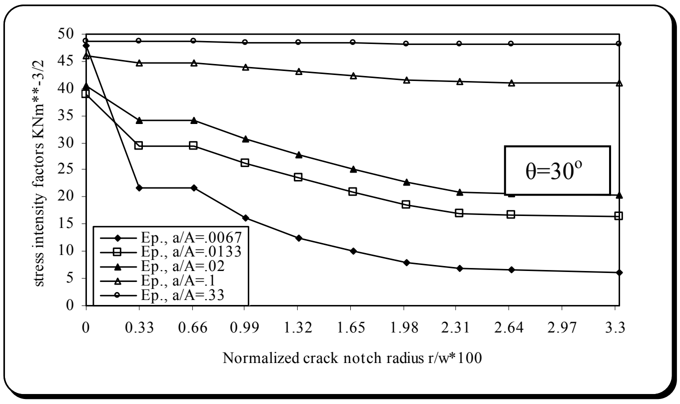



**Fig. 13. Effect of crack notch radius on the stress intensity factor, KII, for a filler material (Epoxy) for different pre-crack lengths and crack notch angles equal to 30°.**

Figure 13 shows the effect of crack notch radius on the shear mode stress intensity factor for a filler material (Epoxy) for different pre-crack lengths and crack notch angles equal to 30°. It is observed that the shear loading mode stress intensity factor is not affected by the crack notch radius for a pre-crack length greater than or equal to 33% of the total crack length, L. Also, the stress intensity factor is decreased as the notch radius increased for a pre-crack length less than 0.33L.

This indicates that the value of shear loading mode stress intensity factor becomes independent whatever the value of crack notch angle and crack notch radius for a pre-crack length greater than or equal to 33% of the total crack length.

### *3.3. Tear Mode Stress Intensity Factor, $K_{III}$*

Figure 14 shows the effect of the pre-crack length on the tear loading mode stress intensity factor for a filler material (Epoxy) for different crack notch angles and sharp notch, *i.e.*, notch radius, r, approximately equal to zero. It is observed that the stress intensity factor, $K_{III}$, increases as the crack notch angle increases. For notch angle less than or equal to 30$^o$ the stress intensity factor, $K_{III}$, decreases as the pre-crack length increases, and for notch angle greater than 30$^o$ the tear mode stress intensity factor, $K_{III}$, increases as the pre-crack length increases reaching a maximum value at a pre-crack length equal to 10% of the total crack length, L, then the stress intensity factor, $K_{III}$, decreases as the pre-crack length increases and remains constant for a pre-crack length greater than or equal to 33% of the total crack length.

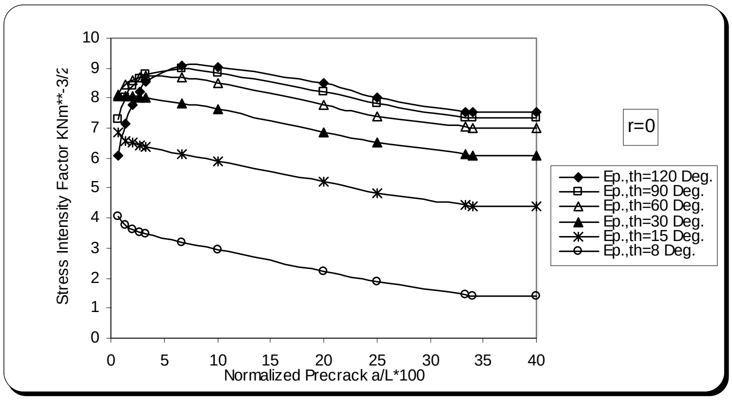



**Fig. 14. Effect of the pre-crack length on the stress intensity factor, KIII, for a filler material (Epoxy) for different crack notch angles and r/w ≈ 0.**

Figure 15 shows the effect of the pre-crack length on the tear loading mode stress intensity factor, $K_{III}$, for a filler material (Epoxy) for different crack notch angles with a blunt notch, r, equal to 0.027 of specimen width, W. The tear mode stress intensity factor is not affected by the crack notch angle. Also, the tear loading mode stress intensity factor increases as the pre-crack length increases reaching a maximum at a pre-crack length less than or equal to 10% of the total crack length. After this value of pre-crack length the stress intensity factor, $K_{III}$, decreases and it remains constant for the pre-crack length greater than or equal to 33% of the total crack length.

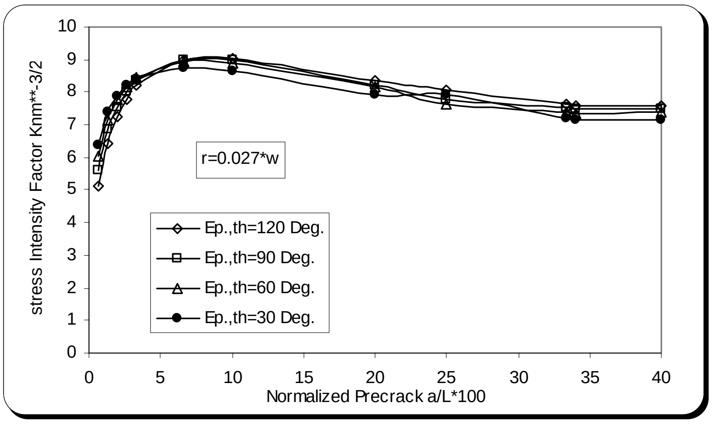



**Fig. 15. Effect of the pre-crack length on the stress intensity factor, KIII, for a filler material (Epoxy) for different crack notch angles and r/w = 0.027.**

Figure 16 shows the effect of the crack notch radius on the tear loading mode stress intensity factor, $K_{III}$, for a filler material (Epoxy) for different pre-crack lengths and crack notch angle equal to 30°.It is observed that for a small pre-crack length, the tear mode stress intensity factor decreases as the notch radius increases. However, in case of a pre-crack greater than or equal to 10% of total crack length, L, the stress intensity factor is not significant by the notch radius.

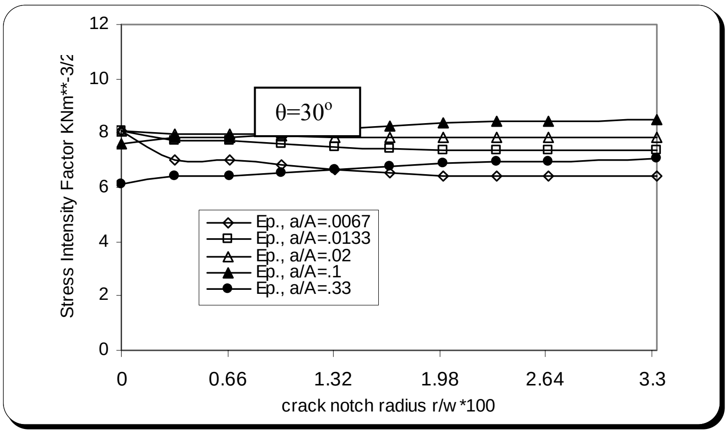



**Fig. 16. Effect of the crack notch radius on the stress intensity factor, KIII, for a filler material (Epoxy) for different pre-crack lengths and crack notch angle equal to 30°.**

## 4. Conclusion Remarks

In the present work, the effect of notch radius, notch angle and pre-crack length on the stress intensity factors for mode I, mode II and mode III have been studied using finite element analysis method for bonded homogeneous materials.

The conclusion consists of three groups; the first group is concerned with stress intensity factor for a specimen under opening mode loading conditions. The second group is concerned with stress intensity factor for a specimen under shear mode loading conditions. The third group is concerned with stress intensity factor for a specimen under tear mode loading conditions.

### *4.1 For Opening Mode Stress Intensity Factor, $K_I$*

- For sharp notch, the opening loading mode stress intensity factor, $K_I$, becomes independent on the crack notch angle beyond a value of $30^\circ$ or below whatever the value of pre-crack length.

- For blunt notch, the stress intensity factor, $K_I$, is little affected by the crack notch angle.

- The stress intensity factor, $K_I$, remains constant for a pre-crack length greater than or equal to 10% L, regardless of the notch radius and notch angle.

### *4.2 For Shear Mode Stress Intensity Factor, $K_{II}$*

- For sharp notch, the shear loading mode stress intensity factor, $K_{II}$, increase as the crack notch angle decreases reaching constant value at an angle less than or equal $15^\circ$ whatever the value of pre-crack length.

- For a blunt notch, the shear loading mode stress intensity factor increases as the pre-crack length increases reaching a maximum value at a pre-crack length, a, greater than or equal to 33.33 % of the total crack length, L. However, for the blunt notch, the shear loading mode stress intensity factor is not affected by the crack notch angle.

- The value of shear loading mode stress intensity factor becomes independent whatever the value of crack notch angle and crack notch radius for a pre-crack length greater than or equal to 33.33 % of the total crack length.

### *4.3. For Tear Mode Stress Intensity Factor, $K_{III}$*

- For sharp notch, the tear loading mode stress intensity factor, $K_{III}$, for bonded homogeneous materials is highly significant by the value of notch angle.

- For blunt notch, the stress intensity factor, $K_{III}$, is not affected by the crack notch angle. However, in case of a pre-crack greater than or equal to 33.33 % of total crack length, L and notch angle equal to $30^o$ the stress intensity factor is not affected by the notch radius.

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# ـديد معامل كثافة الإجهادات لأ اط الكسور الثلاثة باستخدام تقنيات العناصر الصغ ة ثلاثية الأبعاد

**حسن سيد محمد هدية، وسعد محمد رشيد الدوسري**

*قسم هندسة الإنتاج وتصميم النظم الميكانيكية كلية الهندسة جامعة الملك عبدالعزيز - جدة – المملكة العربية السعودية*

*المستخلص*. إن نظام المادتين (Bimaterial systems) موجود في العديد من هياكل المعدات والأدوات الهندسية وتطبيقاتها. لذلك تتم هذه الدارسة على عينات ملصوقة بمادة الأبوكسي لجزئين متماثلين. ولمعرفة سلوك حد إطاقة الكسر في الأجزاء الملصوقة تم دراسة تأثير طول شرخ الكلال الابتدائي على معامل كثافة الإجهاد لأنماط الكسور الثلاثة وذلك لإيجاد الطول المناسب الذي لا يؤثر على قيمة إطاقة حد الكسر. بالإضافة إلى ذلك تم دراسة كل من زاوية حز الشرخ الأصلي ونصف قطره على معامل كثافة الإجهاد لأنماط الكسور الثلاثة لإيجاد القيم المناسبة لتلك العوامل التي لا تؤثر على قيمة إطاقة حد الكسر. وتم استخدام طريقة العناصر المحددة ثلاثية الأبعاد للتحليل العددي للاجهادات في تحديد وتحليل المعاملات المختلفة وتأثيرها على مقدار معامل كثافة الإجهاد وذلك باستخدام برنامج الـ ANSYS التجاري.